Calculus Warm Up # 8-1

1. Locate all the absolute extrema of f(x) on [-1, 2]

$$f(x) = 3x^2 - 6x^3 + 3x$$

2. Find two positive numbers such that their product is 98 and the sum of the first plus twice the second is a minimum. Looking for Calculus, not guess and check!!

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4. $f(x) = x + \dfrac{4}{x^2}$

In Exercises 27–32, find the point(s) guaranteed by the Mean Value Theorem for the indicated interval.

$f'(c) = \frac{f(b)-f(a)}{b-a}$

Function	Interval
27. $f(x) = \frac{2x+3}{3x+2}$	$1 \le x \le 5$

$\frac{-5}{(3c+2)^2} = \frac{f(5)-f(1)}{5-1}$

$f'(x) = \frac{-5}{(3x+2)^2}$

29. $f(x) = x^{2/3}$ $\quad 1 \le x \le 8$

31. $f(x) = x - \frac{1}{x}$ $\quad 1 \le x \le 4$

$f'(x) = 1 + \frac{1}{x^2}$

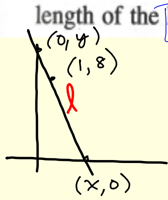
$1 + \frac{1}{c^2} = \frac{f(4)-f(1)}{4-1}$

$=$

33. Can the Mean Value Theorem be applied to the function $f(x) = 1/x^2$ on the interval $[-2, 1]$?

No; f(x) has a discontinuity at x = 0, therefore it does not meet the criteria of the hypothesis.

41. A right triangle in the first quadrant has the coordinate axes as sides, and the hypotenuse passes through the point (1, 8). Find the vertices of the triangle so that the length of the hypotenuse is minimum.



$l = \sqrt{x^2 + y^2}$

min when min of $x^2 + y^2$

so let $g = x^2 + y^2$

Need 2nd relationship so we can replace y

Use slope:

$\frac{y-8}{0-1} = \frac{0-8}{x-1}$

$y = \frac{8}{x-1} + \frac{8(x-1)}{(x-1)}$

$y = \frac{8x}{x-1}$

$g = x^2 + \left(\frac{8x}{x-1}\right)^2$

Now find g', set $= 0$ & solve

$g' = 2x + 2\left(\frac{8x}{x-1}\right)\left[\frac{(x-1)(8) - 8x(1)}{(x-1)^2}\right]$

$g' = \frac{2x(x-1)^3}{(x-1)^3} + \frac{16x(8x-8-8x)}{(x-1)^3}$

$0 = \frac{2x((x-1)^3 - 64)}{(x-1)^3}$

$x = 0 \qquad (x-1)^3 - 64 = 0$

$\boxed{x = 5}$

Vertices

(0,0) (5,0)

& (0,10)

AP Calculus AB
AP Problem Set #1

Our first AP practice. Yellow WS.

(No Calculator)

1. Let f be the function given by $f(x) = x^3 - 7x + 6$.

$0 = x^3 - 7x + 6$

(a) Find the zeros of f.

(b) Write an equation of the line tangent to the graph of f at $x = -1$.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -7 & 6 \\ & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & 0 \end{array}$$

$x^2 + x - 6$

$(x+3)(x-2)$

$-3, 1, 2$

$f(-1) =$

$f'(-1) =$

$y - 12 = -4(x + 1)$

2. Consider the curve defined by $x^2 + xy + y^2 = 27$.

   (a) Write an expression for the slope of the curve at any point (x, y).

   (b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

   (c) Find the points on the curve where the lines tangent to the curve are vertical.

$x + 2y = 0$

a) $\frac{dy}{dx} = -\frac{2x+y}{x+2y}$

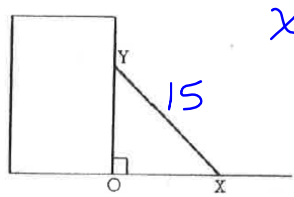
b) $x = \pm 3\sqrt{3}$; both slopes $= -2$
(x-intercepts) (put into $\frac{dy}{dx}$)

$x = \pm\sqrt{27}$

3. A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

   (a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

   (b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.



$x^2 + y^2 = 15^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(9)\frac{1}{2} + 2(12)\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{1}{2}$

when
$x = 9$
$y = 12$

a) $\frac{dy}{dt} = -\frac{3}{8}$ ft/sec

$A = \left(\frac{1}{2}x\right)y$

b) $\frac{dA}{dt} = \frac{21}{16}$ ft²/sec

$\frac{dA}{dt} = \frac{1}{2}x\frac{dy}{dt} + \frac{1}{2}\frac{dx}{dt}\cdot y$

Salmon AP Practice Ch. 4

II. Draw a possible graph of a function f that has ALL of the following properties. Clearly indicate where f has local extrema and points of inflection.

- For $x<0$, $f'(x)>0$ *and* $f''(x)>0$
- $\boxed{f(0)=3,}$ $f'(0)>0$, $f''(0)=0$
- For $0<x<2$, $f'(x)>0$ *and* $f''(x)<0$
- $f'(2)=0$
- For $x>2$, $f'(x)<0$ *and* $f''(x)<0$

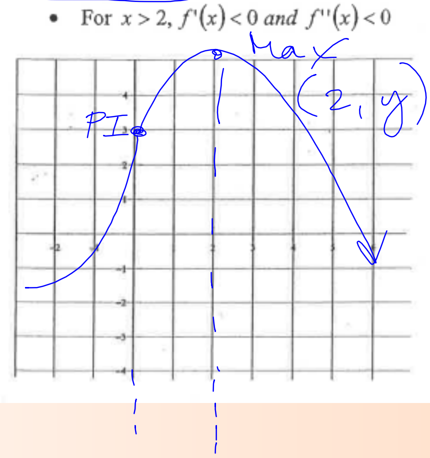
f': + 0 + 2 −

~~poss.~~ extrema Max

f'': + 0 − 2 −

~~poss~~ PI (0,3)

conc. dow



HW:

AP Review Worksheet #1

(due turned in on Thursday)

Test: Tues. and Wed.